Measurement of higher moments of transverse momentum of charged particles in proton-proton collisions
PH 219 Project Group E

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(Dated: December 9, 2020)

In this report we study the characteristics of transverse momentum and look at the trends of standardized and intensive skewness , and standardized variance of p-p collisions at centre of mass energy = 13 Tev generated using Pythia 8 Monte Carlo event generator. Analysis is done for different multiplicity classes. Here STAR method is used for skewness analysis. We have made use of the STAR method for analysis of central moments which can be found in [1]. We also see that the data of the p-p collisions have a positive skewness for the $\langle p_t \rangle$ distribution as was expected as per arguments provided in [1]. The ROOT macros that were used for analysis can be found here.

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I. INTRODUCTION

The data provided is generated with Pythia 8 Monte Carlo event generator.
Number of events : 2 million
Collisions System : p + p at centre of mass energy 13 TeV.
In ultra relativistic collisions mean transverse momentum denoted by $\langle p_t \rangle$ fluctuates event to event.
While one may expect that $\langle p_t \rangle$ would have a gaussian distribution due to the central limit theorem one can see as refelected from the study of the data that all of them seem to have a considerable positive skew and we shall explore the trend of this with different multiplicity classes. Formulae used according to STAR definition-
(All the averaging is being done over number of events in each multiplicity class).

$$\langle\langle p_t \rangle\rangle_{STAR} \equiv \left\langle \frac{\sum_{i=1}^{N_{ch}} p_i}{N_{ch}} \right\rangle \tag{1}$$

$$\langle \Delta p_i \Delta p_j \rangle_{STAR} \equiv \left\langle \frac{\sum_{i,j\neq i}(p_i - \langle\langle p_t \rangle\rangle)(p_j - \langle\langle p_t \rangle\rangle)}{N_{ch}(N_{ch}-1)} \right\rangle \tag{2}$$

$$\langle \Delta p_i \Delta p_j \Delta p_k \rangle_{STAR} \equiv \left\langle \frac{\sum_{i,j\neq i,k\neq j,i}(p_i - \langle\langle p_t \rangle\rangle)(p_j - \langle\langle p_t \rangle\rangle)(p_k - \langle\langle p_t \rangle\rangle)}{N_{ch}(N_{ch}-1)(N_{ch}-2)} \right\rangle \tag{3}$$

For calculating these quantities we use the following formulae as described in Appendix A of [1].

$$Q_n = \sum_{i=1}^{N_{ch}} (p_i)^n \tag{4}$$

$$\sum_{i,j\neq i} p_i p_j = (Q_1)^2 - Q_2 \tag{5}$$

$$\sum_{i,j\neq i,k\neq j,i} p_i p_j p_k = (Q_1)^3 - 3Q_2 Q_1 + 2Q_3 \tag{6}$$

Using this we rewrite equations (1) to (3) as

$$\langle\langle p_t \rangle\rangle_{STAR} \equiv \left\langle \frac{Q_1}{N_{ch}} \right\rangle \tag{7}$$

$$\langle \Delta p_i \Delta p_j \rangle_{STAR} \equiv \left\langle \frac{(Q_1)^2 - Q_2}{N_{ch}(N_{ch}-1)} \right\rangle - \left\langle \frac{Q_1}{N_{ch}} \right\rangle^2 \tag{8}$$

$$\langle \Delta p_i \Delta p_j \Delta p_k \rangle_{STAR} \equiv \left\langle \frac{(Q_1)^3 - 3Q_2 Q_1 + 2Q_3}{N_{ch}(N_{ch}-1)(N_{ch}-2)} \right\rangle - 3\left\langle \frac{(Q_1)^2 - Q_2}{N_{ch}(N_{ch}-1)} \right\rangle \left\langle \frac{Q_1}{N_{ch}} \right\rangle + 2\left\langle \frac{Q_1}{N_{ch}} \right\rangle^3 \tag{9}$$

Using this we wish to find skewness (central third moment) and also the standardized variance (two particle correlator) of the distributions of $\langle p_T \rangle$ for different multiplicities. These can be expressed using equations (7) to (9).

$$\gamma_{p_t} = \frac{\langle \Delta p_i \Delta p_j \Delta p_k \rangle}{\langle \Delta p_i \Delta p_j \rangle^{3/2}} \tag{10}$$

$$\Gamma_{p_t} = \frac{\langle \Delta p_i \Delta p_j \Delta p_k \rangle \langle\langle p_t \rangle\rangle}{\langle \Delta p_i \Delta p_j \rangle^2} \tag{11}$$

$$\sigma_{standardized} = \frac{\sqrt{\langle \Delta p_i \Delta p_j \rangle}}{\langle\langle p_t \rangle\rangle} \tag{12}$$

γ_{p_t} is the expression for standardized skewness and Γ_{p_t} is the expression for intensive skewness. Both of these are dimensionless quantities. $\sigma_{standardized}$ is the variance that is normalized to the mean of $\langle p_t \rangle$ and is dimensionless. One can find these values using STAR method and obtaining the values using equations (7) to (9).

II. EXPERIMENTAL OBSERVATIONS

A. Plots of distribution of p_T

The following are plots of distribution of transverse momentum in logarithmic scale. This has been attempted to fit with an exponential distribution (the red line). The data points are represented with the blue histogram. The graph is frequency of value versus transverse momentum value in Gev/c^2

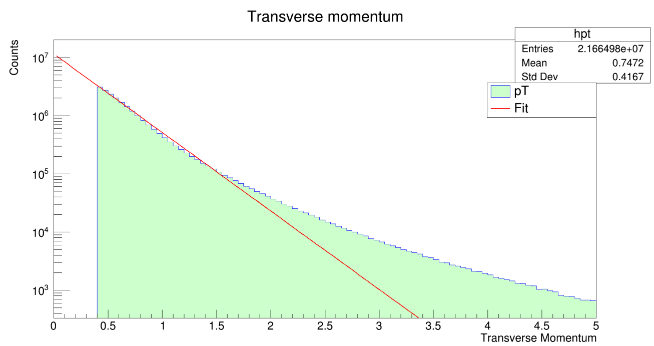

FIG. 1. p_T distribution for 0-20 multiplicity

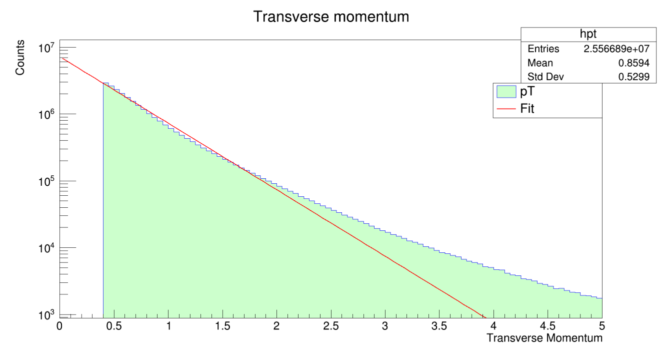

FIG. 2. p_T distribution for 20-40 multiplicity

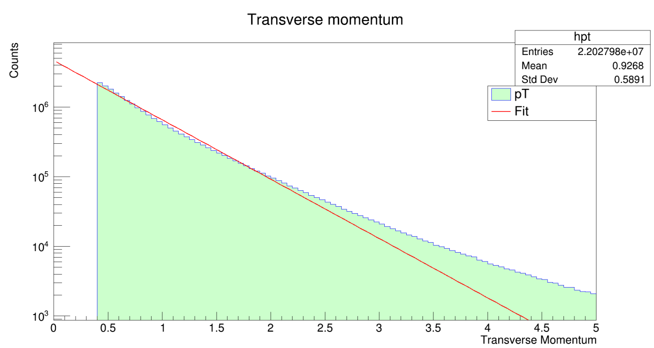

FIG. 3. p_T distribution for 40-60 multiplicity

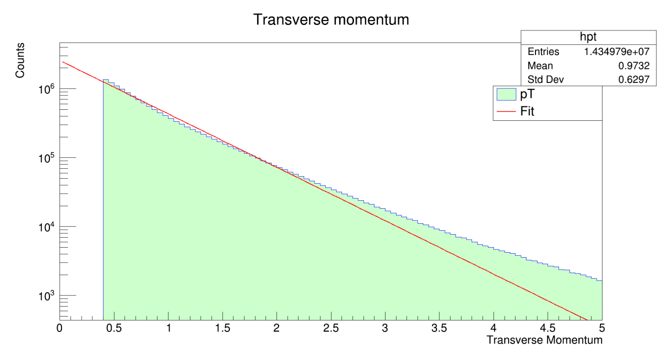

FIG. 4. p_T distribution for 60-80 multiplicity

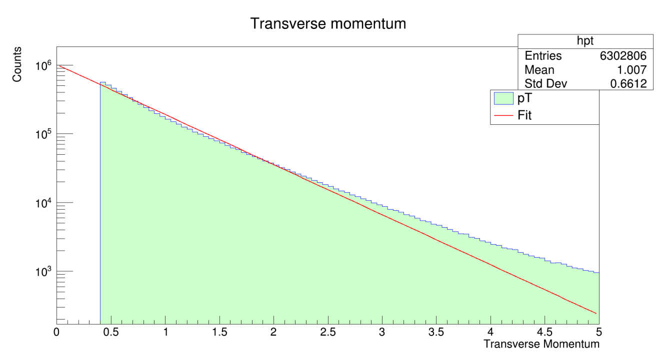

FIG. 5. p_T distribution for 80-100 multiplicity

B. Plots of distribution of $\langle p_T \rangle$

The following are plots of distribution of averaged transverse momentum over N_{ch} (ntrack) for each event in logarithmic scale. This has been attempted to fit with a Gaussian distribution (the red curve) since central limit theorem will follow when using sample averages. The data points are represented by the blue histogram. The graph is frequency of value versus average transverse momentum value in Gev/c^2.

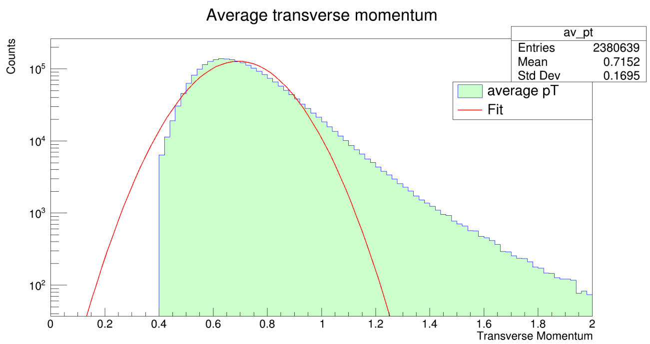

FIG. 6. $\langle p_T \rangle$ distribution for 0-20 multiplicity

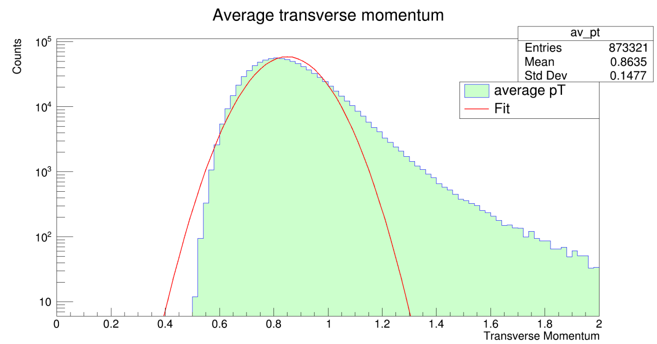

FIG. 7. $\langle p_T \rangle$ distribution for 20-40 multiplicity

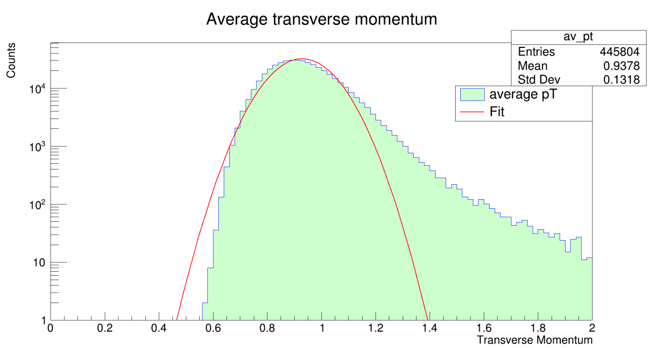

FIG. 8. $\langle p_T \rangle$ distribution for 40-60 multiplicity

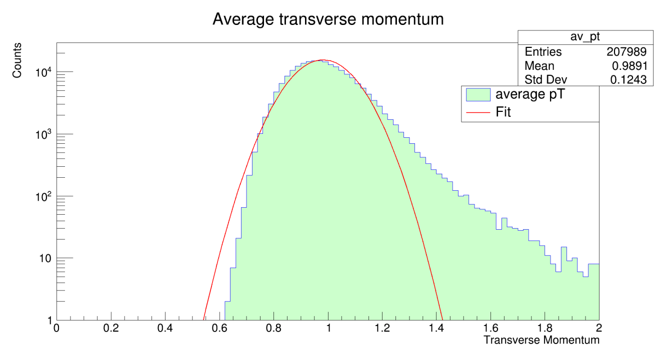

FIG. 9. $\langle p_T \rangle$ distribution for 60-80 multiplicity

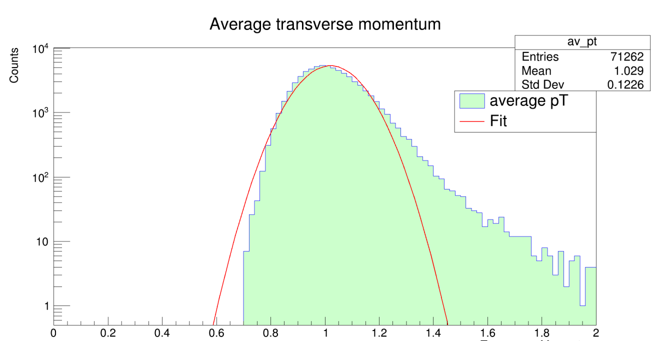

FIG. 10. $\langle p_T \rangle$ distribution for 80-100 multiplicity

C. Central moments for $\langle p_T \rangle$

Using the equations (10) to (12) we plot the variation for the central moments with the centrality (multiplicity). All of these are taken with $|\eta| < 2.5$. The plots connect data points in the manner that if the point is at 30 multiplicity that implies $20 - 40$ multiplicity. The two particle correlator is essentially the expression $\sigma_{stadardized}$ which is in equation (11).

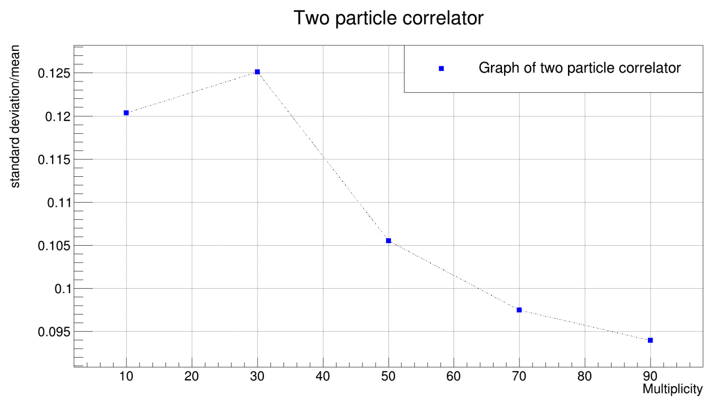

FIG. 11. Two particle correlator versus multiplicity

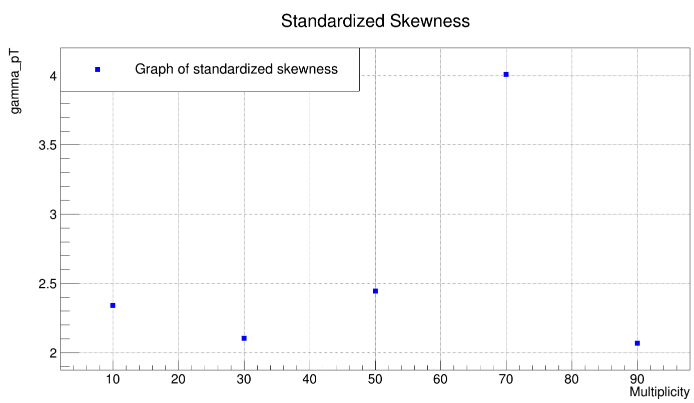

FIG. 12. Standardized skewness versus multiplicity

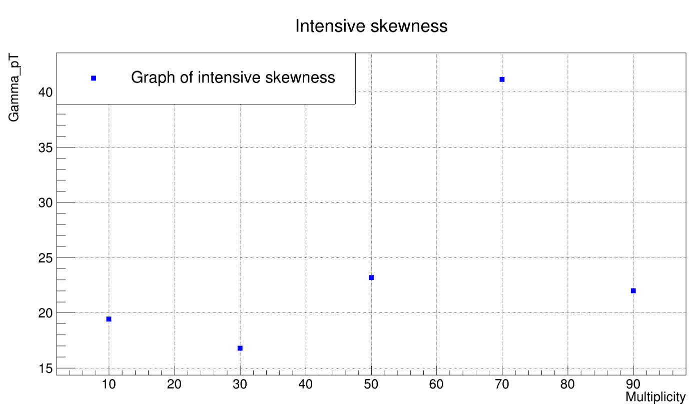

FIG. 13. Intensive skewness versus multiplicity

III. SUMMARY

We have studied the generated data for p-p collisions and there are some things which we can infer from the data. The p_T values follow an exponential distribution approximately which gives us the idea that higher momentum values are much rarer to come by and so is a fairly natural choice. One must note that the values do seem to diverge from expectation as the value of p_T increases and there is a cutoff below which there are no readings which can be infered as particles with this little momentum fail to reach the detector in the first place.
The distribution for average momentum (averaged over N_{ch}) should be close to a Gaussian due to the central limit theorem and while it certainly does seem to somewhat look Gaussian there is a long tail for each histogram and this tail is essentially what gives this a considerable positive skewness which reflects in our later calculation for γ_{p_t} and Γ_{p_t}.
Now when it comes to calculating the values for the central moments the values do not seem to have a solid correlation but one can roughly conclude that the skewness increases with the multiplicity. Another way to look at this is that skewness would decrease if we have more entries since central limit theorem would hold more strongly then and we can see that the entries do decrease with increasing multiplicity. The two particle correlator however would be expected to increase since it is essentially root of sample variance divided by average of sample mean and the former decreases with increase in number of entries while the latter is independent of the same and so we would expect an upward trend. We are getting a roughly downward trend but the ranges here are very close by so a conclusive judgement cannot be made and there may be statistical errors.

LINK TO CODE REPOSITORY

You can find the ROOT macros that were used in the following link:
https://github.com/mahadevans2432/PH219-ROOT-project

REFERENCES

[1] Giuliano Giacalone et al. *Skewness of mean transverse momentum fluctuations in heavy-ion collisions*. 2020. arXiv: 2004.09799 [nucl-th].