BELL'S MEASURE IN CLASSICAL OPTICAL COHERENCE

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Introduction

- Bell States - entangled qubits, correlated measurements
- Classical Optics analogue with parity and polarisation DOFs ('qubits')
- True randomness (incoherence) vs coupled DOFs (individually appear random)
- Possible to share the coherence 'resource' between different DOFs
- A simple classical implementation of a quantum computer

Polarization & Spatial Parity

- Polarization is either H or V and parity is defined as spatial parity of beam along x axis as even and odd
- $\mathbf{J} = \begin{bmatrix} E_{\text{He}} & E_{\text{Ho}} & E_{\text{Ve}} & E_{\text{Vo}} \end{bmatrix}^T$
- Coherency matrix $\mathbf{G} = \langle \mathbf{J}^* \mathbf{J}^T \rangle \equiv$ density matrix formalism
- Purity of quantum states $\equiv$ coherency of beam
- An apparent deficit of coherence when DoFs are coupled.

Experimental Setup

- Basic Idea: encode the action of each physical piece of equipment (HWP, SLM and PS-SLM) as a time independent matrix operator A.
- $J_{\text{out}} = AJ_{\text{in}}$ and $G = \langle J^* J^T \rangle \implies G_{\text{out}} = A^* G_{\text{in}} A^T$

Properties of these matrix operators

- Matrix A unique only upto global phase factor $e^{\iota\psi}$ but G_{out} is completely unique.
- Completely incoherent beam - G is multiple of identity, each A should be unitary.
- A_{HWP} and A_{SLM} commute.

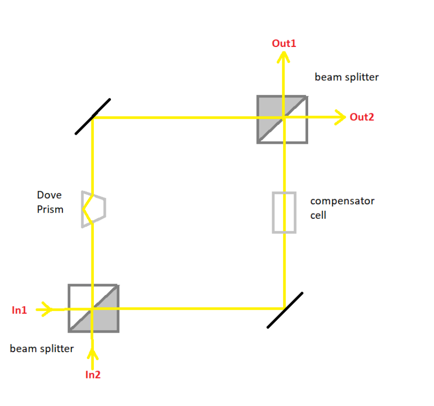
Experimental Setup

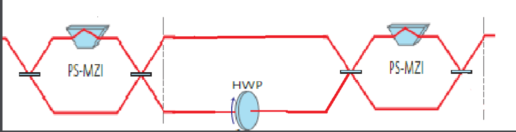
$$A_{HWP}\left(\frac{\theta}{2}\right) = \begin{pmatrix} -\cos\frac{\theta}{2} & 0 & -\sin\frac{\theta}{2} & 0 \\ 0 & -\cos\frac{\theta}{2} & 0 & -\sin\frac{\theta}{2} \\ -\sin\frac{\theta}{2} & 0 & \cos\frac{\theta}{2} & 0 \\ 0 & -\sin\frac{\theta}{2} & 0 & \cos\frac{\theta}{2} \end{pmatrix}$$

$$A_{SLM}\left(\frac{\phi}{2}\right) = \begin{pmatrix} \cos\frac{\phi}{2} & i\sin\frac{\phi}{2} & 0 & 0 \\ i\sin\frac{\phi}{2} & \cos\frac{\phi}{2} & 0 & 0 \\ 0 & 0 & \cos\frac{\phi}{2} & i\sin\frac{\phi}{2} \\ 0 & 0 & i\sin\frac{\phi}{2} & \cos\frac{\phi}{2} \end{pmatrix}$$

$$A_{PS-SLM}\left(\frac{\phi}{2}\right) = \begin{pmatrix} \cos\frac{\phi}{2} & i\sin\frac{\phi}{2} & 0 & 0 \\ i\sin\frac{\phi}{2} & \cos\frac{\phi}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Experimental Setup





Dove Prism - Acts like $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ operator on parity

The three experiments

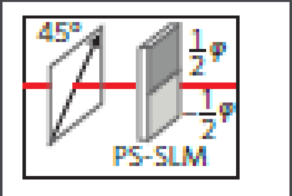


Figure: Experiment A

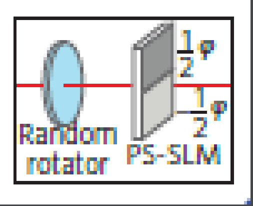


Figure: Experiment B

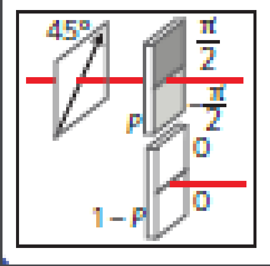


Figure: Experiment C

All three images have been sourced from the main paper (Bell's measure in classical optical coherence, K.H. Kagalwala et al, Nature Photonics).

Analysis Stage

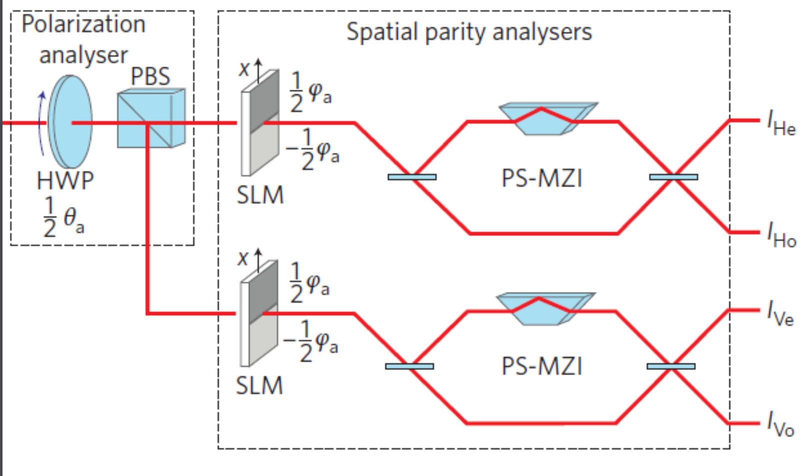


Figure: Analysis Stage

Analysis Stage

Introducing bell's measure

$$B = |C(\theta_a, \varphi_a) + C(\theta_a, \varphi'_a) + C(\theta'_a, \varphi_a) - C(\theta'_a, \varphi'_a)| \tag{1}$$

Correlation function defined as $C(\theta_a, \varphi_a) = \sum c_{\text{pol}} c_{\text{par}} P_{\text{pol,par}} = P_{\text{He}} - P_{\text{H0}} - P_{\text{Ve}} + P_{\text{Vo}}$.
Define accessible degrees of coherence

$$S_{\text{pol}} = \frac{D_{\text{pol}}^2}{2} + \left(\frac{B_{max}}{2\sqrt{2}}\right)^2, \; S_{\text{par}} = \frac{D_{\text{par}}^2}{2} + \left(\frac{B_{max}}{2\sqrt{2}}\right)^2 \tag{2}$$

Now we can quantify coupling and explain the "deficit" in coherence

$$S = \frac{4}{3}\left(\text{Tr}(\mathbf{G}^2) - \frac{1}{4}\right)$$

Experiments: Experiment A

- Coherent beam with coupled degrees of freedom.
- $J = \frac{1}{\sqrt{2}} \begin{bmatrix} \cos\left(\frac{\phi}{2}\right) & \iota \sin\left(\frac{\phi}{2}\right) & 1 & 0 \end{bmatrix}^T$
- $D_{pol} = D_{par} = \left|\cos\left(\frac{\phi}{2}\right)\right|$
- $C\left(\theta_a, \phi_a\right) = \sin\left(\theta_a\right)\cos\left(\frac{\phi}{2} + \phi_a\right) - \frac{1}{2}\cos\left(\theta_a\right)\cos\left(\phi_a\right) + \frac{1}{2}\cos\left(\theta_a\right)\cos\left(\phi + \phi_a\right)$

Experiments: Experiment A Plots

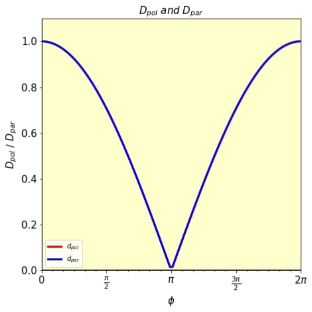


Figure: D_{pol} and D_{par} vs ϕ

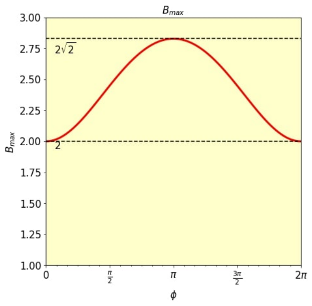


Figure: B_{max} vs ϕ

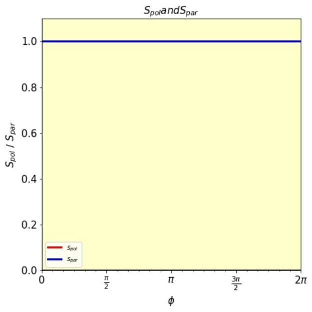


Figure: S_{pol} and S_{par} vs ϕ

Experiments: Experiment A Contour Plots

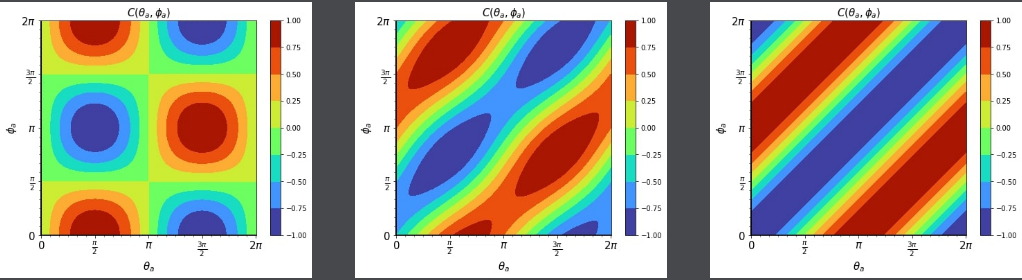


Figure: Correlation Function vs θ_a and ϕ_a for $\phi = 0$, $\phi = \frac{\pi}{2}$ and $\phi = \pi$ from left to right.

Experiments: Experiment B

- Partially coherent beam with coupled degrees of freedom.
- J is similar to that in experiment A, except that the polarisation has been scrambled using polarizers, without affecting parity.
- $D_{pol} = 0$ and $D_{par} = \left|\cos\left(\frac{\phi}{2}\right)\right|$
- $C(\theta_a, \phi_a) = \frac{1}{2}\cos(\theta_a)\left[\cos(\phi + \phi_a) - \cos(\phi_a)\right]$

Experiments: Experiment B Plots

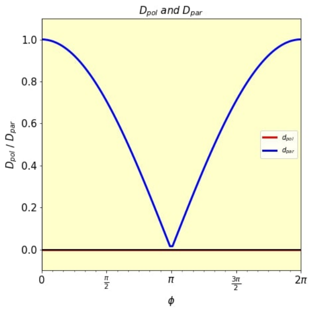


Figure: D_{pol} and D_{par} vs ϕ

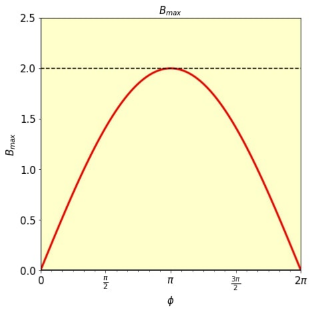


Figure: B_{max} vs ϕ

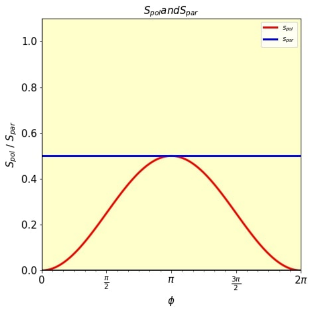


Figure: S_{pol} and S_{par} vs ϕ

Experiments: Experiment B Contour Plots

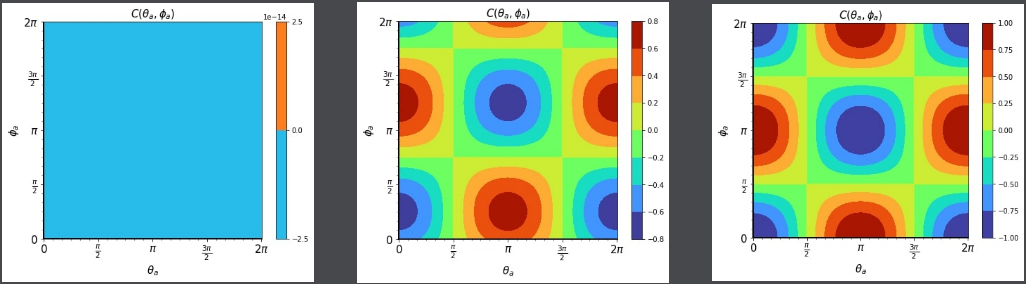


Figure: Correlation Function vs θ_a and ϕ_a for $\phi = 0$, $\phi = \frac{\pi}{2}$ and $\phi = \pi$ from left to right.

Experiments: Experiment C

- Mixture of 2 beams.
- J is $\frac{1}{\sqrt{2}}\begin{bmatrix}0 & \iota & 1 & 0\end{bmatrix}^T$ with probability P and $\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 0 & 1 & 0\end{bmatrix}^T$ with probability $1 - P$
- $D_{pol} = D_{par} = 1 - P$
- $C\left(\theta_a, \phi_a\right) = (1 - P)\cos\left(\theta_a\right)\sin\left(\phi_a\right) - P\cos(\theta_a - \phi_a)$

Experiments: Experiment C Plots

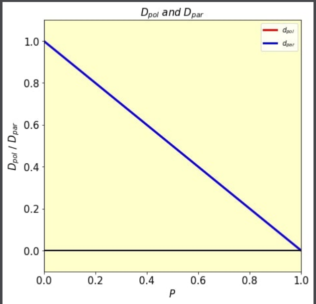


Figure: D_{pol} and D_{par} vs P

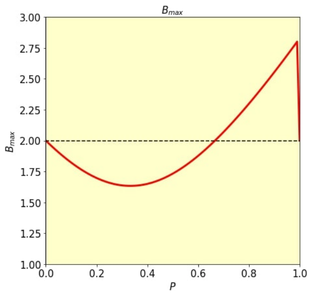


Figure: B_{max} vs P

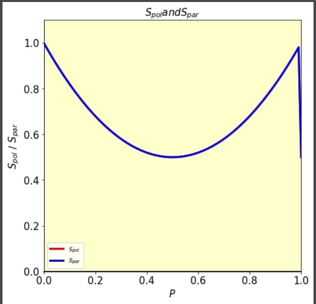


Figure: S_{pol} and S_{par} vs P

Experiments: Experiment C Contour Plots

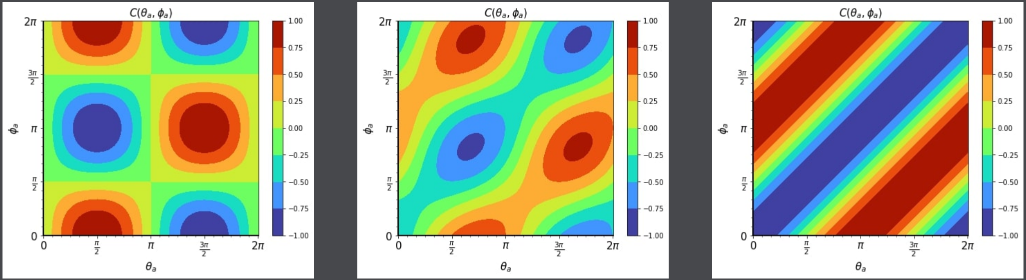


Figure: Correlation Function vs θ_a and ϕ_a for $P = 0$, $P = 0.5$ and $P = 1$ from left to right.

Reduced Representation of Two Binary DoFs

- We cannot have all combinations of S, S_{pol}, S_{par} inside the unit cube but only the ones which satisfy Bell's Inequality.
- For a given value of S, we get a concave-sided triangular area within which the values of S_{pol} and S_{par} must be contained.
- For given values of S_{pol} and S_{par}, we can predict S within a narrow range by drawing a line segment parallel to S-axis inside the figure.
- For $S < \frac{1}{3}$, for an uncoupled beam, $S_{pol} = 0$ or $S_{par} = 0$

Reduced Representation of Two Binary DoFs

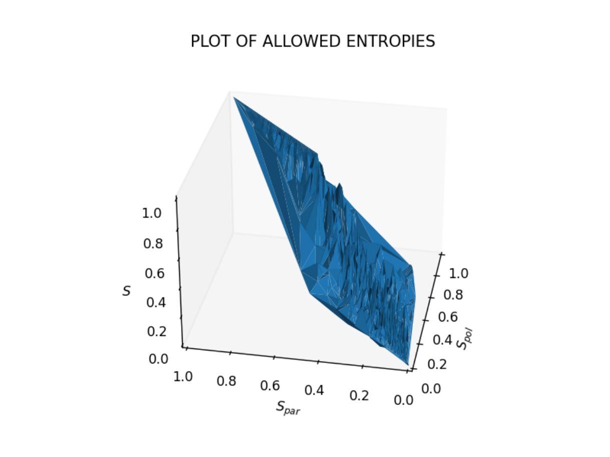


Figure: Plot of Allowed Entropies

Quantum Circuits

All the previous experiments can be encoded into quantum circuits which reproduce the results

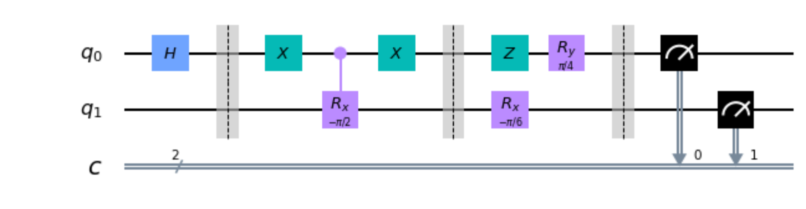


Figure: Circuit for experiment A

Quantum Circuits

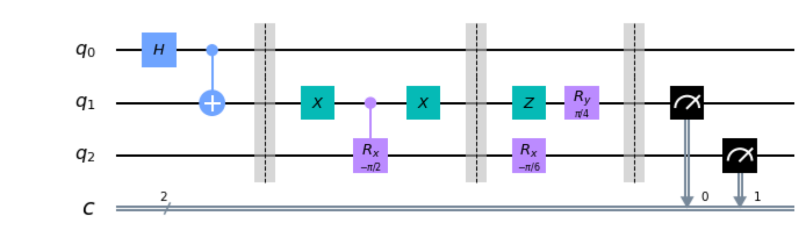


Figure: Circuit for experiment B

Quantum Circuits

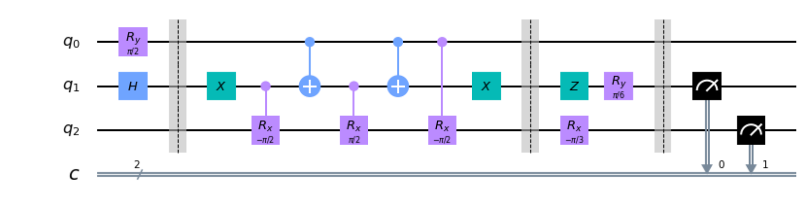


Figure: Circuit for experiment C

Quantum Circuits: Universality

We can show that we have minimal set of operations required for universality

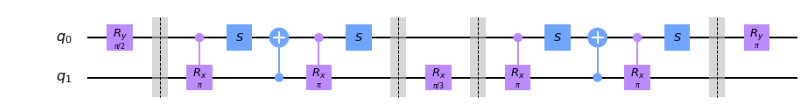


Figure: Arbitrary U3 on polarization

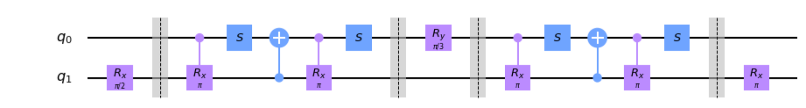


Figure: Arbitrary U3 on parity

Conclusion

- We can indeed quantify coupling using bell's measure
- An interesting link between quantum theory and classical coherence
- Since $B_{max} > 2$ in certain cases, we cannot use hidden variables to write $C(\theta_a, \phi_a) = \int d\lambda \rho(\lambda) C_{pol}(\theta_a, \lambda) C_{par}(\phi_a, \lambda)$
- Universality for two qubits proven (can be extended to 3) potential applications for quantum computers using just beams of light.

Thank You !